AMERICAN UNIVERSITY OF BEIRUT

MATHEMATICS 235 FINAL EXAMINATION

February 1, 2001

Time = 1 Hour and 30 Minutes

1. For a simple linear regression model which is given by the following: $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$, where $i = 1, ..., n$ and ϵ_i are IID with mean 0 and variance σ^2. Show that the least squares estimate of β_1, $\hat{\beta}_1 = \sum(x_i - \bar{x})(y_i - \bar{y}) / \sum(x_i - \bar{x})^2$, is an unbiased estimator of β and find its variance.

2. Manatees are large creatures that live in shallow water along the coast of Florida. Many manatees are injured or killed each year by power boats. Here are data on manatees killed and powerboat registration (in thousands of boats) in Florida for the period 1977 to 1990:

Year	Powerboat registrations	Manatees Killed
1977	447	13
1978	460	21
1979	481	24
1980	498	16
1981	513	24
1982	512	20
1983	526	15
1984	559	34
1985	585	33
1986	614	33
1987	645	39
1988	675	43
1989	711	50
1990	719	47

Here is a partial output from the Minitab regression command for these data, with boat registration as the explanatory variable and manatees killed as the response variable:

over →

Predictor	Coef	Stdev	t-ratio	p
Constant	-41.430	7.412	-5.59	0.000
Boats	0.12486	0.01290	9.68	0.000

$s = 4.276$ and $R^2 = 88.6\%$.

(a) Construct the ANOVA table for this problem.

(b) What is the meaning of R^2 here?

(c) Explain how to find a 95% confidence interval for the number of killed manatees if the number of registered boats is 700.

3. You are given the following multiple regression model:

$$y_i = \beta_0 + \beta_1 * i + \beta_2 * i^2 + \epsilon_i \qquad (1)$$

where $i = 1, \ldots, n$ and ϵ_i are IID $N(0, \sigma^2)$. σ^2 is assumed to be an unknown constant.

(a) Formulate the model in equation (1) in a matrix form, i.e., $\underline{Y} = X\underline{\beta} + \underline{\epsilon}$

(b) Compute $X^T X$. (Hint: You may make use of the following identities: $\sum i = n(n+1)/2$, $\sum i^2 = n(n+1)(2n+1)/6$, $\sum i^3 = [n(n+1)/2]^2$, and $\sum i^4 = n(n+1)(2n+1)(3n^2+3n-1)/30$)

(c) Can you compute $\underline{\hat{\beta}}$, the least squares estimate of $\underline{\beta}$ without inverting the product matrix $X^T X$. If yes explain how?